Core Mathematics C4 For Edexcel
Advanced Level

Paper E
Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration.

Full marks may be obtained for answers to ALL questions.

The booklet 'Mathematical Formulae and Statistical Tables', available from Edexcel, may be used.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner.
Answers without working may gain no credit.

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1. A curve has equation

$$4x^2 + 3y^2 = 12.$$

(*a*) Find the *x*-coordinates of the two points on the curve at which the *y*-coordinate is 1. **(2)**

(*b*) Find the gradient of the curve at these two points. **(4)**

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2. (*a*) Expand $(8 + x)^{\frac{1}{3}}$ in ascending powers of x, up to and including the term in x^2. **(4)**

(*b*) Show that, if m^3 and higher powers of m are neglected,

$$\left(8 + 3m + m^2\right)^{\frac{1}{3}} = 2 + \frac{1}{4}m + \frac{5}{96}m^2.$$

(3)

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3. The parametric equations of a curve are

$$x = \cos\theta, \quad y = \frac{1}{2}\sin 2\theta, \quad 0 \leq \theta \leq 2\pi.$$

(*a*) Find an expression for $\frac{dy}{dx}$ in terms of θ and hence find the gradient of the curve at the point where $\theta = \frac{\pi}{6}$. **(2)**

(*b*) Show that the equation of the tangent to the curve at the point where $\theta = \frac{\pi}{6}$ is $4y + 4x = 3\sqrt{3}$. **(3)**

(*c*) Show that the cartesian equation of the curve is $y^2 = x^2\left(1 - x^2\right)$. **(3)**

4.

Figure 1

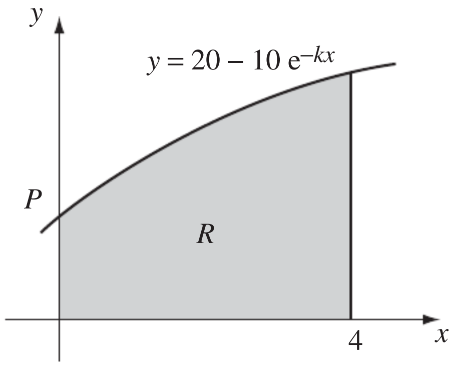

Figure 1 shows part of the curve C with equation

$$y = 20 - 10\,e^{-kx}.$$

(*a*) Write down the coordinates of the point P where C crosses the y-axis. **(1)**

(*b*) The gradient of C at the point P is 5. Show that $k = \frac{1}{2}$. **(3)**

(*c*) Find the area of the region R which is bounded by C, the positive axes and the line $x = 4$. **(5)**

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5. At time t minutes after being switched on, the temperature of an oven $\theta°C$ is given by

$$\theta = 300 - 270\,e^{-0.05t}$$

(*a*) Find θ when $t = 0$. **(1)**

(*b*) Find the value which θ approaches after a long time. **(2)**

(*c*) Find the time taken to reach a temperature of $200°C$. **(3)**

(*d*) Find the rate at which the temperature is increasing when $t = 2$. **(3)**

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6. (*a*) Express $\dfrac{2}{(1-x)(2-x)}$ in partial fractions. **(2)**

(*b*) Show that, for small values of x,

$$\frac{2}{(1-x)(2-x)} \simeq 1 + \frac{3}{2}x + ax^2,$$

where a is to be found. **(6)**

(*c*) Show that $\displaystyle\int_0^{\frac{1}{2}} \frac{2}{(1-x)(2-x)}\, dx = 2\ln\frac{3}{2}$ **(5)**

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7. Relative to a fixed origin O, the point A has position vector $\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$, the point B has position vector $5\mathbf{i} + 3\mathbf{j} + \mathbf{k}$, and the point C has position vector $-\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}$.

(*a*) Show that the cosine of angle ABC is $\dfrac{3}{\sqrt{46}}$. **(4)**

(*b*) Find the exact value of the area of triangle ABC. **(4)**

The point D has position vector $4\mathbf{i} - 2\mathbf{j} - 12\mathbf{k}$.

(*c*) Show that AC is parallel to OD. **(2)**

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8. (*a*) Use the substitution $t = 2x + 1$ to show that

$$\int_0^1 \frac{x}{(2x+1)^2}\, dx = \frac{1}{4}\left(\ln 3 - \frac{2}{3}\right)$$ **(7)**

(*b*) Use integration by parts to find the exact value of $\displaystyle\int_1^{e} x^2 \ln x \, dx$. **(6)**

END **TOTAL 75 MARKS**